

Package ‘mig’

July 22, 2025

Type Package

Title Multivariate Inverse Gaussian Distribution

Version 2.0

Description Provides utilities for estimation for the multivariate inverse Gaussian distribution of Minami (2003) <[doi:10.1081/STA-120025379](https://doi.org/10.1081/STA-120025379)>, including random vector generation and explicit estimators of the location vector and scale matrix. The package implements kernel density estimators discussed in Belzile, Desgagnes, Genest and Ouimet (2024) <[doi:10.48550/arXiv.2209.04757](https://doi.org/10.48550/arXiv.2209.04757)> for smoothing multivariate data on half-spaces.

BugReports <https://github.com/lbelzile/mig/issues>

Imports statmod, TruncatedNormal (>= 2.3), Rcpp (>= 1.0.12)

Depends R (>= 2.10)

Suggests numDeriv, tinytest, knitr, rmarkdown, minqa

LinkingTo Rcpp, RcppArmadillo

Encoding UTF-8

LazyData true

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VignetteBuilder knitr

RoxygenNote 7.3.2

NeedsCompilation yes

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Repository CRAN

Date/Publication 2025-04-08 22:50:02 UTC

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an	<i>Threshold selection for bandwidth</i>
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Description

Automated thresholds selection for the robust likelihood cross validation. The cutoff is based on the covariance matrix of the sample data.

Usage

an(x)

Arguments

x matrix of observations

Value

cutoff for robust selection

dmig	<i>Multivariate inverse Gaussian distribution</i>
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Description

The density of the MIG model is

$$f(\mathbf{x} + \mathbf{a}) = (2\pi)^{-d/2} \boldsymbol{\beta}^\top \boldsymbol{\xi} |\boldsymbol{\Omega}|^{-1/2} (\boldsymbol{\beta}^\top \mathbf{x})^{-(1+d/2)} \exp \left\{ -\frac{(\mathbf{x} - \boldsymbol{\xi})^\top \boldsymbol{\Omega}^{-1} (\mathbf{x} - \boldsymbol{\xi})}{2\boldsymbol{\beta}^\top \mathbf{x}} \right\}$$

for points in the d-dimensional half-space $\{\mathbf{x} \in \mathbb{R}^d : \boldsymbol{\beta}^\top (\mathbf{x} - \mathbf{a}) \geq 0\}$

Usage

```
dmig(x, xi, Omega, beta, shift, log = FALSE)

rmig(n, xi, Omega, beta, shift, method = c("invsim", "bm"), timeinc = 0.001)

pmig(q, xi, Omega, beta, log = FALSE, method = c("sov", "mc"), B = 10000L)
```

Arguments

x	n by d matrix of quantiles
xi	d vector of location parameters ξ , giving the expected value
Omega	d by d positive definite scale matrix Ω
beta	d vector β defining the half-space through $\beta^\top \xi > 0$
shift	d translation for the half-space a
log	logical; if TRUE, returns log probabilities
n	number of observations
method	string; one of inverse system (invsim, default), Brownian motion (bm)
timeinc	time increment for multivariate simulation algorithm based on the hitting time of Brownian motion, default to 1e-3.
q	n by d matrix of quantiles
B	number of Monte Carlo replications for the SOV estimator

Details

Observations are generated using the representation as the first hitting time of a hyperplane of a correlated Brownian motion.

Value

for dmig, the (log)-density
 for rmig, an n vector if d=1 (univariate) or an n by d matrix if d > 1
 an n vector of (log) probabilities

Author(s)

Frederic Ouimet (bm), Leo Belzile (invsim)
 Leo Belzile

Examples

```
# Density evaluation
x <- rbind(c(1, 2), c(2,3), c(0,-1))
beta <- c(1, 0)
xi <- c(1, 1)
Omega <- matrix(c(2, -1, -1, 2), nrow = 2, ncol = 2)
```

```

dmig(x, xi = xi, Omega = Omega, beta = beta)
# Random number generation
d <- 5L
beta <- rexp(d)
xi <- rexp(d)
Omega <- matrix(0.5, d, d) + diag(d)
samp <- rmig(n = 1000, beta = beta, xi = xi, Omega = Omega)
stopifnot(isTRUE(all(samp %*% beta > 0)))
mle <- fit_mig(samp, beta = beta, method = "mle")
set.seed(1234)
d <- 2L
beta <- runif(d)
Omega <- rWishart(n = 1, df = 2*d, Sigma = matrix(0.5, d, d) + diag(d))[,,1]
xi <- rexp(d)
q <- mig::rmig(n = 10, beta = beta, Omega = Omega, xi = xi)
pmig(q, xi = xi, beta = beta, Omega = Omega)

```

fit_mig

Fit multivariate inverse Gaussian distribution

Description

Fit multivariate inverse Gaussian distribution

Usage

```
fit_mig(x, beta, method = c("mle", "mom"), shift)
```

Arguments

x	n by d matrix of quantiles
beta	d vector β defining the half-space through $\beta^T \xi > 0$
method	string, one of mle for maximum likelihood estimation, or mom for method of moments.
shift	d translation for the half-space a

Value

a list with components:

- xi: estimate of the expectation or location vector
- Omega: estimate of the scale matrix

geomagnetic *Magnetic storms*

Description

Absolute magnitude of 373 geomagnetic storms lasting more than 48h with absolute magnitude (dst) larger than 100 in 1957-2014.

Format

a vector of size 373

Note

For a detailed article presenting the derivation of the Dst index, see <http://wdc.kugi.kyoto-u.ac.jp/dstdir/dst2/onDst>

Source

Aki Vehtari

References

World Data Center for Geomagnetism, Kyoto, M. Nose, T. Iyemori, M. Sugiura, T. Kamei (2015), *Geomagnetic Dst index*, doi:10.17593/14515-74000.

hsgauss_kdens *Gaussian kernel density estimator on half-space*

Description

Given a data matrix over a half-space defined by beta, compute an homeomorphism to \mathbb{R}^d and perform kernel smoothing based on a Gaussian kernel density estimator, taking each turn an observation as location vector.

Usage

```
hsgauss_kdens(x, newdata, Sigma, beta, log = TRUE, ...)
```

Arguments

x	n by d matrix of quantiles
newdata	matrix of new observations at which to evaluated the kernel density
Sigma	scale matrix
beta	d vector β defining the half-space through $\beta^T \xi > 0$
log	logical; if TRUE, returns log probabilities
...	additional arguments, currently ignored

Value

a vector containing the value of the kernel density at each of the newdata points

kdens_bandwidth	<i>Optimal scale matrix for kernel density estimation</i>
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Description

Given an n sample from a multivariate distribution on the half-space defined by $\{\mathbf{x} \in \mathbb{R}^d : \beta^\top \mathbf{x} > 0\}$, the function computes the bandwidth (type="isotropic") or scale matrix that minimizes the asymptotic mean integrated squared error away from the boundary. The latter depend on the true unknown density, which is replaced by the kernel density or a MIG distribution evaluated at the maximum likelihood estimator. The integral or the integrated squared error are obtained by Monte Carlo integration with N simulations

Usage

```
kdens_bandwidth(
  x,
  beta,
  shift,
  family = c("mig", "hsgauss", "tnorm"),
  method = c("amise", "lcv", "lscv", "rlcv"),
  type = c("isotropic", "diag", "full"),
  approx = c("kernel", "mig", "tnorm"),
  transformation = c("none", "scaling", "spherical"),
  N = 10000L,
  buffer = 0,
  maxiter = 2000L,
  ...
)
```

Arguments

x	an n by d matrix of observations
beta	d vector defining the half-space
shift	location vector for translating the half-space. If missing, defaults to zero
family	distribution for smoothing, either mig for multivariate inverse Gaussian, tnorm for truncated normal on the half-space and hsgauss for the Gaussian smoothing after suitable transformation.
method	estimation criterion, either amise for the expression that minimizes the asymptotic integrated squared error, lcv for likelihood (leave-one-out) cross-validation, lscv for least-square cross-validation or rlcvc for robust cross validation of Wu (2019)

type	string indicating whether to compute an isotropic model or estimate the optimal scale matrix via optimization
approx	string; distribution to approximate the true density function $f(x)$; either kernel for the kernel estimator evaluated at the sample points (except for method="amise", which isn't supported), mig for multivariate inverse Gaussian with the method of moments or tnorm for the multivariate truncated Gaussian evaluated by maximum likelihood.
transformation	string for optional scaling of the data before computing the bandwidth. Either standardization to unit variance scaling, spherical transformation to unit variance and zero correlation (spherical), or none (default).
N	integer number of simulations for Monte Carlo integration
buffer	double indicating the buffer from the half-space
maxiter	integer; max number of iterations in the call to optim.
...	additional parameters, currently ignored

Value

a d by d scale matrix

References

Wu, X. (2019). Robust likelihood cross-validation for kernel density estimation. *Journal of Business & Economic Statistics*, 37(4), 761–770. doi:10.1080/07350015.2018.1424633 Bowman, A.W. (1984). An alternative method of cross-validation for the smoothing of density estimates, *Biometrika*, 71(2), 353–360. doi:10.1093/biomet/71.2.353 Rudemo, M. (1982). Empirical choice of histograms and kernel density estimators. *Scandinavian Journal of Statistics*, 9(2), 65–78. <http://www.jstor.org/stable/4615859>

mig_kdens

Multivariate inverse Gaussian kernel density estimator

Description

Given a matrix of new observations, compute the density of the multivariate inverse Gaussian mixture defined by assigning equal weight to each component where ξ is the location parameter.

Usage

```
mig_kdens(x, newdata, Omega, beta, log = FALSE)
```

Arguments

x	n by d matrix of quantiles
newdata	matrix of new observations at which to evaluate the kernel density
Omega	d by d positive definite scale matrix Ω
beta	d vector β defining the half-space through $\beta^\top \xi > 0$
log	logical; if TRUE, returns log probabilities

Value

value of the (log)-density at newdata

mle_truncgauss	<i>Maximum likelihood estimation of truncated Gaussian on half space</i>
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Description

Given a data matrix and a vector of linear constraints for the half-space, we compute the maximum likelihood estimator of the location and scale matrix

Usage

```
mle_truncgauss(xdat, beta)
```

Arguments

xdat	matrix of observations
beta	vector of constraints defining the half-space

Value

a list with location vector loc and scale matrix scale

proj_hs	<i>Orthogonal projection matrix onto the half-space</i>
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Description

The orthogonal projection matrix P has unit determinant and transforms an n by d matrix by taking $x * P$. The components of the first column vector of the resulting matrix are strictly positive.

Usage

```
proj_hs(beta, inv = FALSE)
```

Arguments

beta	vector defining the half-space
inv	logical; if TRUE, return the inverse matrix

Value

a d by d orthogonal projection matrix

tnorm_kdens	<i>Truncated Gaussian kernel density estimator</i>
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Description

Given a data matrix over a half-space defined by beta, compute the log density of the asymmetric truncated Gaussian kernel density estimator, taking in turn an observation as location vector.

Usage

```
tnorm_kdens(x, newdata, Sigma, beta, log = TRUE, ...)
```

Arguments

x	n by d matrix of quantiles
newdata	matrix of new observations at which to evaluate the kernel density
Sigma	scale matrix
beta	d vector β defining the half-space through $\beta^\top \xi > 0$
log	logical; if TRUE, returns log probabilities
...	additional arguments, currently ignored

Value

a vector containing the value of the kernel density at each of the newdata points

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