Package 'BLA'

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Type Package

```
Title Boundary Line Analysis
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Description Fits boundary line models to datasets as proposed by Webb
      (1972) <doi:10.1080/00221589.1972.11514472> and makes statistical
      inferences about their parameters. Provides additional tools for
      testing datasets for evidence of boundary presence and selecting
      initial starting values for model optimization prior to fitting the
      boundary line models. It also includes tools for conducting post-hoc
      analyses such as predicting boundary values and identifying the most
      limiting factor (Miti, Milne, Giller, Lark (2024)
      <doi:10.1016/j.fcr.2024.109365>). This ensures a comprehensive
      analysis for datasets that exhibit upper boundary structures.
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Description

This function fits a boundary model to the upper bounds of a scatter plot of x and y based on the binning method. The data are first divided into equal sized sections in the x-axis and a boundary point in each section is selected based on a set criteria (e.g. 0.90, 0.95 or 0.99 percentile of y among other criteria). A model is then fitted to the resulting boundary points by the least squares method. This is done using optimization procedure and hence requires some starting guess parameters for the proposed model.

Usage

```
blbin(x,y,bins, model="explore", equation=NULL, start, tau=0.95,
    optim.method="Nelder-Mead", xmin=min(bound$x), xmax=max(bound$x),plot=TRUE,
    bp_col="red", bp_pch=16, bl_col="red", lwd=1,line_smooth=1000,...)
```

Arguments

x A numeric vector of values for the independent variable.

y A numeric vector of values for the response variable.

A numeric vector of length 3 or 4 that determines the size of sections. The first and second values give the range of the data to be binned while the third and fourth values give the width of the bins and the step size respectively. If only three values are provided, the step size is assumed to be equal to bin width.

Selects the functional form of the boundary line. It includes "explore" as default, "blm" for linear model, "lp" for linear plateau model, "mit" for the Mitscherlich model, "schmidt" for the Schmidt model, "logistic" for logistic model, "logistic" for logistic model, "logistic" for logistic model, "double-logistic" for the double logistic model, "qd" for quadratic model and the "trapezium" for the trapezium model. The "explore" is used to check the position of boundary points in each bin so that the correct model can be applied. For custom models, set model = "other".

A custom model function writen in the form of an R function. Applies only when argument model="other", else it is NULL.

A numeric vector of initial starting values for optimization in fitting the boundary model. Its length and arrangement depend on the suggested model:

- For the "blm" model, it is a vector of length 2 arranged as intercept and slope.
- For the "lp" model, it is a vector of length 3 arranged as intercept, slope and maximum response.
- For the "logistic" and "inv-logistic" models, it is a vector of length 3 arranged as the scaling parameter, shape parameter and maximum response.
- For the "logisticND" model proposed by Nelder (1961), it is a vector of length 3 arranged as the scaling parameter, shape parameter and maximum response.
- For the "double-logistic" model, it is a vector of length 6 arranged as the scaling parameter one, shape parameter one, maximum response, scaling parameter two and shape parameter two.
- For the "qd" model, it is a vector of length 3 arranged as constant, linear coefficient and quadratic coefficient.
- For the "trapezium" model, it is a vector of length 3 arranged as intercept one, slope one, maximum response, intercept two and slope two.
- For the "mit" model, it is a vector of length 3 arranged as the intercept, shape parameter and the maximum response.
- For the "schmidt" model, it is a vector of length 3 arranged as scaling parameter, shape parameter (x-value at maximum response) and maximum response.

A percentile value (0-1) that represents the boundary point within each bin (default is tau = 0.95).

model

bins

equation

start

tau

optim.method	Describes the method used to optimize the model as in the optim() function. The methods include "Nelder-Mead", "BFGS", "CG", "L-BFGS-B", "SANN" and "Brent".
xmin	Numeric value that describes the minimum x value to which the boundary line is to be fitted (default is $min(x)$).
xmax	A numeric value that describes the maximum x value to which the boundary line is to be fitted (default is $max(x)$). $xmin$ and $xmax$ determine the subset of the data set used to fit boundary model.
plot	If TRUE, a plot is part of the output. If FALSE, plot is not part of output (default is TRUE).
bp_col	Selects the color of the boundary points.
bp_pch	Point character as pch of the plot() function. It controls the shape of the boundary points on plot (bp_pch = 16 as default).
bl_col	Colour of the boundary line.
lwd	Determines the thickness of the boundary line on the plot (default is 1).
line_smooth	Parameter that describes the smoothness of the boundary line. (default is 1000). The higher the value, the smoother the line.
	Additional graphical parameters as in the par() function.

Details

Some inbuilt models are available for the blbin() function. The "explore" option for the argument model generates a plot showing the location of the boundary points selected by the binning procedure. This helps to identify which model type is suitable to fit as a boundary line. The suggest model forms are as follows:

Linear model ("blm")

$$y = \beta_1 + \beta_2 x$$

where β_1 is the intercept and β_2 is the slope.

2. Linear plateau model ("lp")

$$y = \min(\beta_1 + \beta_2 x, \beta_0)$$

where β_1 is the intercept , β_2 is the slope and β_0 is the maximum response.

3. The logistic ("logistic") and inverse logistic ("inv-logistic") models

$$y = \frac{\beta_0}{1 + e^{\beta_2(\beta_1 - x)}}$$

$$y = \beta_0 - \frac{\beta_0}{1 + e^{\beta_2(\beta_1 - x)}}$$

where β_1 is a scaling parameter , β_2 is a shape parameter and β_0 is the maximum response.

4. Logistic model ("logisticND") (Nelder (2009))

$$y = \frac{\beta_0}{1 + (\beta_1 \times e^{-\beta_2 x})}$$

where β_1 is a scaling parameter, β_2 is a shape parameter and β_0 is the maximum response.

5. Double logistic model ("double-logistic")

$$y = \frac{\beta_{0,1}}{1 + e^{\beta_2(\beta_1 - x)}} - \frac{\beta_{0,2}}{1 + e^{\beta_4(\beta_3 - x)}}$$

where β_1 is a scaling parameter one, β_2 is shape parameter one, $\beta_{0,1}$ and $\beta_{0,2}$ are the maximum response, β_3 is a scaling parameter two and β_4 is a shape parameter two.

6. Quadratic model ("qd")

$$y = \beta_1 + \beta_2 x + \beta_3 x^2$$

where β_1 is a constant, β_2 is a linear coefficient and β_3 is the quadratic coefficient.

7. Trapezium model ("trapezium")

$$y = \min(\beta_1 + \beta_2 x, \beta_0, \beta_3 + \beta_4 x)$$

where β_1 is the intercept one, β_2 is the slope one, β_0 is the maximum response, β_3 is the intercept two and β_3 is the slope two.

8. Mitscherlich model ("mit")

$$y = \beta_0 - \beta_1 * \beta_2^x$$

where β_1 is the intercept, β_2 is a shape parameter and β_0 is the maximum response.

9. Schmidt model ("schmidt")

$$y = \beta_0 + \beta_1 (x - \beta_2)^2$$

where β_1 is ascaling parameter, β_2 is a shape parameter (x-value at maximum response) and β_0 is the maximum response .

10. Custom model ("other") This option allows you to create your own model form using the function function(). The custom model should be assigned to the argument equation. Note that the parameters for the custom model should be a and b for a two parameter model; a, b and c for a three parameter model; a, b, c and d for a four parameter model and so on.

The function blbin() utilities the optimization procedure of the optim() function to determine the model parameters. There is a tendency for optimization algorithms to settle at a local optimum. To remove the risk of settling for local optimum parameters, it is advised that the function is run using several starting values and the results with the smallest error (residue mean square) can be taken as a representation of the global optimum.

The common errors encountered due to poor start values

- 1. function cannot be evaluated at initial parameters
- 2. initial value in 'vmmin' is not finite

Value

A list of length 5 consisting of the fitted model, equation form, parameters of the boundary line, the residue mean square and the boundary points. Additionally, a graphical representation of the boundary line on the scatter plot is produced.

Author(s)

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References

Casanova, D., Goudriaan, J., Bouma, J., & Epema, G. (1999). Yield gap analysis in relation to soil properties in direct-seeded flooded rice.

Nelder, J.A. 1961. The fitting of a generalization of the logistic curve. Biometrics 17: 89-110.

Phillips, B.F. & Campbell, N.A. 1968. A new method of fitting the von Bertelanffy growth curve using data on the whelk. Dicathais, Growth 32: 317–329.

Schmidt, U., Thöni, H., & Kaupenjohann, M. (2000). Using a boundary line approach to analyze N2O flux data from agricultural soils. Nutrient Cycling in Agro-ecosystems, 57, 119-129.

Examples

ble_profile

Likelihood profile for various measurement error values

Description

Estimates the standard deviation of measurement error (sign) of the response variable, an input of the cbvn() function, when a measured value is not available (Lark & Milne, 2016). sigh is fixed at each of a set of values in turn, and remaining parameters are estimated conditional on sigh by maximum likelihood. The maximized likelihoods for the sequence of values constitutes a likelihood profile. The value of sigh where the profile is maximized is selected.

Usage

Arguments

data	A dataframe with two numeric columns, independent (x) and dependent (y) vari-
	ables respectively.

sigh A vector of the suggested standard deviations of the measurement error values.

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model

Selects the functional form of the boundary line. It includes "blm" for linear model, "lp" for linear plateau model, "mit" for the Mitscherlich model, "schmidt" for the Schmidt model, "logistic" for logistic model, "logisticND" for logistic model proposed by Nelder (1961), "inv-logistic" for the inverse logistic model, "double-logistic" for the double logistic model, "qd" for quadratic model and the "trapezium" for the trapezium model. For custom models, set model = "other".

equation

A custom model function writen in the form of an R function. Applies only when argument model="other", else it is NULL.

start

A numeric vector of initial starting values for optimization in fitting the boundary model. Its length and arrangement depend on the suggested model:

- For the "blm" model, it is a vector of length 7 arranged as the intercept, the slope, mean of x, mean of y, standard deviation of x, standard deviation of y and the correlation of x and y.
- For the "1p" model, it is a vector of length 8 arranged as the intercept, the slope, the maximum or plateau response, mean of x, mean of y, standard deviation of x, standard deviation of y and the correlation of x and y.
- For the "mit" model, it is a vector of length 8 arranged as the intercept, shape parameter, the maximum or plateau response, mean of x, mean of y, standard deviation of x, standard deviation of y and the correlation of x and y.
- For the "logistic", "inv-logistic" and "logisticND" models, it is a vector of length 8 arranged as scaling parameter, shape parameter, the maximum or plateau value, mean of x, mean of y, standard deviation of x, standard deviation of y and the correlation of x and y.
- For the "double-logistic" model, it is a vector of length 11 arranged as scaling parameter, shape parameter, maximum response, maximum response, scaling parameter two, shape parameter two, mean of x, mean of y, standard deviation of x, standard deviation of y and the correlation of x and y.
- For the "trapezium" model, it is a vector of length 10 arranged as intercept one, slope one, maximum response, intercept two, slope two, mean of x, mean of y, standard deviation of x, standard deviation of y and the correlation of x and y.
- For the "qd" model, it is a vector of length 8 arranged as a constant, linear coefficient, quadratic coefficient, mean of x, mean of y, standard deviation of x, standard deviation of y and the correlation of x and y.
- For the "schmidt" model, it is a vector of length 8 arranged the scaling parameter, shape parameter (x-value at maximum response), maximum response, mean of x, mean of y, standard deviation of x, standard deviation of y and the correlation of x and y.

UpLo

Selects the type of boundary. "U" fits the upper boundary and "L" fits the lower boundary.

optim.method

Describes the method used to optimize the model as in the optim() function. The methods include "Nelder-Mead", "BFGS", "CG", "L-BFGS-B", "SANN" and "Brent".

plot

If TRUE, a plot is part of the output. If FALSE, plot is not part of output (default is TRUE).

Details

Some inbuilt models are available for the cbvn() function. The suggest model forms are as follows:

1. Linear model ("blm")

$$y = \beta_1 + \beta_2 x$$

where β_1 is the intercept and β_2 is the slope.

2. Linear plateau model ("lp")

$$y = \min(\beta_1 + \beta_2 x, \beta_0)$$

where β_1 is the intercept, β_2 is the slope and β_0 is the maximum response.

3. The logistic ("logistic") and inverse logistic ("inv-logistic") models

$$y = \frac{\beta_0}{1 + e^{\beta_2(\beta_1 - x)}}$$

$$y = \beta_0 - \frac{\beta_0}{1 + e^{\beta_2(\beta_1 - x)}}$$

where β_1 is a scaling parameter , β_2 is a shape parameter and β_0 is the maximum response.

4. Logistic model ("logisticND") (Nelder (1961))

$$y = \frac{\beta_0}{1 + (\beta_1 \times e^{-\beta_2 x})}$$

where β_1 is a scaling parameter, β_2 is a shape parameter and β_0 is the maximum response.

5. Double logistic model ("double-logistic")

$$y = \frac{\beta_{0,1}}{1 + e^{\beta_2(\beta_1 - x)}} - \frac{\beta_{0,2}}{1 + e^{\beta_4(\beta_3 - x)}}$$

where β_1 is a scaling parameter one, β_2 is shape parameter one, $\beta_{0,1}$ and $\beta_{0,2}$ are the maximum response, β_3 is a scaling parameter two and β_4 is a shape parameter two.

6. Quadratic model ("qd")

$$y = \beta_1 + \beta_2 x + \beta_3 x^2$$

where β_1 is a constant, β_2 is a linear coefficient and β_3 is the quadratic coefficient.

7. Trapezium model ("trapezium")

$$y = \min(\beta_1 + \beta_2 x, \beta_0, \beta_3 + \beta_4 x)$$

where β_1 is the intercept one, β_2 is the slope one, β_0 is the maximum response, β_3 is the intercept two and β_3 is the slope two.

8. Mitscherlich model ("mit")

$$y = \beta_0 - \beta_1 * \beta_2^x$$

where β_1 is the intercept, β_2 is a shape parameter and β_0 is the maximum response.

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9. Schmidt model ("schmidt")

$$y = \beta_0 + \beta_1 (x - \beta_2)^2$$

where β_1 is ascaling parameter, β_2 is a shape parameter (x-value at maximum response) and β_0 is the maximum response .

The function ble_profile() utilities the optimization procedure of the optim() function to determine the model parameters. There is a tendency for optimization algorithms to settle at a local optimum. To remove the risk of settling for local optimum parameters, it is advised that the function is run using several starting values and the results with the largest likelihood can be taken as a representation of the global optimum.

The common errors encountered due to poor start values

- 1. function cannot be evaluated at initial parameters
- 2. initial value in 'vmmin' is not finite

Value

A list of length 2 containing the suggested standard deviations of measurement error values and the corresponding log-likelihood values. additionally, a likelihood profile plot (log-likelihood against the standard deviation of measurement error) is produced.

Author(s)

Chawezi Miti <chawezi.miti@nottingham.ac.uk>

References

Lark, R. M., & Milne, A. E. (2016). Boundary line analysis of the effect of water filled pore space on nitrous oxide emission from cores of arable soil. European Journal of Soil Science, 67, 148-159.

Nelder, J.A. 1961. The fitting of a generalization of the logistic curve. Biometrics 17: 89–110.

Examples

```
x<-evapotranspiration$`ET(mm)`
y<-evapotranspiration$`yield(t/ha)`
data<-data.frame(x,y)
start<-c(0.5,0.02,289.6,2.4,83.7,1.07,0.287)
sigh <- c(0.6,0.7,0.8,0.9)
ble_profile(data,start=start,model = "blm", sigh = sigh)</pre>
```

10 blqr

blqr

Boundary line model determination using quantile regression

Description

This function fits a boundary model to the upper bounds of a scatter plot of x and y by estimating the conditional quantile (0-1) of the response variable, y, across values of the predictor variables, x. This is achieved using optimization procedure and hence requires some starting guess parameters of a proposed model.

Usage

Arguments

x A numeric vector of values for the independent variable.

y A numeric vector of values for the response variable.

model

Selects the functional form of the boundary line. It includes "blm" for linear model, "lp" for linear plateau model, "mit" for the Mitscherlich model, "schmidt" for the Schmidt model, "logistic" for logistic model, "logisticND" for logistic model proposed by Nelder (1961), "inv-logistic" for the inverse logistic model, "double-logistic" for the double logistic model, "qd" for quadratic model and the "trapezium" for the trapezium model. For custom models, set model = "other".

equation

A custom model function writen in the form of an R function. Applies only when argument model="other", else it is NULL.

start

A numeric vector of initial starting values for optimization in fitting the boundary model. Its length and arrangement depend on the suggested model:

- For the "blm" model, it is a vector of length 2 arranged as intercept and slope.
- For the "lp" model, it is a vector of length 3 arranged as intercept, slope and maximum response.
- For the "logistic" and "inv-logistic" models, it is a vector of length 3 arranged as the scaling parameter, shape parameter and maximum response.
- For the "logisticND" model proposed by Nelder (1961), it is a vector of length 3 arranged as the scaling parameter, shape parameter and maximum response.
- For the "double-logistic" model, it is a vector of length 6 arranged as the scaling parameter one, shape parameter one, maximum response, maximum response, scaling parameter two and shape parameter two.
- For the "qd" model, it is a vector of length 3 arranged as constant, linear coefficient and quadratic coefficient.

• For the "trapezium" model, it is a vector of length 3 arranged as intercept one, slope one, maximum response, intercept two and slope two.

• For the "mit" model, it is a vector of length 3 arranged as the intercept, shape parameter and the maximum response.

• For the "schmidt" model, it is a vector of length 3 arranged as scaling parameter, shape parameter (x-value at maximum response) and maximum response.

tau The quantile value (0-1) that represents the boundary (default is tau = 0.95).

optim.method Describes the method used to optimize the model as in the optim() function. The methods include "Nelder-Mead", "BFGS", "CG", "L-BFGS-B", "SANN" and

The methods method relater medu, bros, co, L-bros-b, sann an

"Brent".

xmin Numeric value that describes the minimum x value to which the boundary line

is to be fitted (default is min(x)).

xmax A numeric value that describes the maximum x value to which the boundary

line is to be fitted (default is max(x)). xmin and xmax determine the subset of

the data set used to fit boundary model.

plot If TRUE, a plot is part of the output. If FALSE, plot is not part of output (default

is TRUE).

line_col Selects the color of the boundary line.

lwd Determines the thickness of the boundary line on the plot (default is 1).

line_smooth Parameter that describes the smoothness of the boundary line. (default is 1000).

The higher the value, the smoother the line.

... Additional graphical parameters.

Details

Some inbuilt models are available for the blqr() function. The suggest model forms are as follows:

1. Linear model ("blm")

$$y = \beta_1 + \beta_2 x$$

where β_1 is the intercept and β_2 is the slope.

2. Linear plateau model ("lp")

$$y = \min(\beta_1 + \beta_2 x, \beta_0)$$

where β_1 is the intercept, β_2 is the slope and β_0 is the maximum response.

3. The logistic ("logistic") and inverse logistic ("inv-logistic") models

$$y = \frac{\beta_0}{1 + e^{\beta_2(\beta_1 - x)}}$$

$$y = \beta_0 - \frac{\beta_0}{1 + e^{\beta_2(\beta_1 - x)}}$$

where β_1 is a scaling parameter, β_2 is a shape parameter and β_0 is the maximum response.

4. Logistic model ("logisticND") (Nelder (1961))

$$y = \frac{\beta_0}{1 + (\beta_1 \times e^{-\beta_2 x})}$$

blqr

where β_1 is a scaling parameter, β_2 is a shape parameter and β_0 is the maximum response.

Double logistic model ("double-logistic")

$$y = \frac{\beta_{0,1}}{1 + e^{\beta_2(\beta_1 - x)}} - \frac{\beta_{0,2}}{1 + e^{\beta_4(\beta_3 - x)}}$$

where β_1 is a scaling parameter one, β_2 is a shape parameter one, $\beta_{0,1}$ and $\beta_{0,2}$ are the maximum response, β_3 is a scaling parameter two and β_4 is a shape parameter two.

6. Quadratic model ("qd")

$$y = \beta_1 + \beta_2 x + \beta_3 x^2$$

where β_1 is a constant, β_2 is a linear coefficient and β_3 is the quadratic coefficient.

7. Trapezium model ("trapezium")

$$y = \min(\beta_1 + \beta_2 x, \beta_0, \beta_3 + \beta_4 x)$$

where β_1 is the intercept one, β_2 is the slope one, β_0 is the maximum response, β_3 is the intercept two and β_3 is the slope two.

8. Mitscherlich model ("mit")

$$y = \beta_0 - \beta_1 * \beta_2^x$$

where β_1 is the intercept, β_2 is a shape parameter and β_0 is the maximum response.

9. Schmidt model ("schmidt")

$$y = \beta_0 + \beta_1 (x - \beta_2)^2$$

where β_1 is ascaling parameter, β_2 is a shape parameter (x-value at maximum response) and β_0 is the maximum response .

10. Custom model ("other") This option allows you to create your own model form using the function function(). The custom model should be assigned to the argument equation. Note that the parameters for the custom model should be a and b for a two parameter model; a, b and c for a three parameter model; a, b, c and d for a four parameter model and so on.

The function blbin() utilities the optimization procedure of the optim() function to determine the model parameters. There is a tendency for optimization algorithms to settle at a local optimum. To remove the risk of settling for local optimum parameters, it is advised that the function is run using several starting values and the results with the smallest error (weighted residue sum square) can be taken as a representation of the global optimum.

The common errors encountered due to poor start values

- 1. function cannot be evaluated at initial parameters
- 2. initial value in 'vmmin' is not finite

Value

A list of length 5 consisting of the fitted model, equation form, parameters of the boundary line, the weighted residue sum square. Additionally, a graphical representation of the boundary line on the scatter plot is produced.

Author(s)

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References

Cade, B. S., & Noon, B. R. (2003). A gentle introduction to quantile regression for ecologists. Frontiers in Ecology and the Environment, 1(8), 412-420.

Nelder, J.A. 1961. The fitting of a generalization of the logistic curve. Biometrics 17: 89–110.

Phillips, B.F. & Campbell, N.A. 1968. A new method of fitting the von Bertelanffy growth curve using data on the whelk. Dicathais, Growth 32: 317–329.

Schmidt, U., Thöni, H., & Kaupenjohann, M. (2000). Using a boundary line approach to analyze N2O flux data from agricultural soils. Nutrient Cycling in Agroecosystems, 57, 119-129.

Examples

bolides

Boundary line determination technique

Description

This function selects upper bounding points of a scatter plot of x and y based on the boundary line determination technique proposed by Schnug et al. (1995). A model is then fitted to the resulting boundary points by the least squares method. This is done using optimization procedure and hence requires some starting values for the model parameters for the proposed model.

Usage

Arguments

x A numeric vector of values for the independent variable.

y A numeric vector of values for the response variable.

model

Selects the functional form of the boundary line. It includes "explore" as default, "blm" for linear model, "lp" for linear plateau model, "mit" for the Mitscherlich model, "schmidt" for the Schmidt model, "logistic" for logistic model, "logisticND" for logistic model proposed by Nelder (1961), "inv-logistic" for the inverse logistic model, "double-logistic" for the double logistic model, "qd" for quadratic model and the "trapezium" for the trapezium model. The "explore" is used to check the position of boundary points so that the correct model can be applied. For custom models, set model = "other".

equation

A custom model function writen in the form of an R function. Applies only when argument model="other", else it is NULL.

A numeric vector of initial starting values for optimization in fitting the boundary model. Its length and arrangement depend on the suggested model:

- For the "blm" model, it is a vector of length 2 arranged as intercept and slope.
- For the "lp" model, it is a vector of length 3 arranged as intercept, slope and maximum response.
- For the "logistic" and "inv-logistic" models, it is a vector of length 3 arranged as the scaling parameter, shape parameter and maximum response.
- For the "logisticND" model proposed by Nelder (1961), it is a vector of length 3 arranged as the scaling parameter, shape parameter and maximum response.
- For the "double-logistic" model, it is a vector of length 6 arranged as the scaling parameter one, shape parameter one, maximum response, maximum response, scaling parameter two and shape parameter two.
- For the "qd" model, it is a vector of length 3 arranged as constant, linear coefficient and quadratic coefficient.
- For the "trapezium" model, it is a vector of length 3 arranged as intercept one, slope one, maximum response, intercept two and slope two.
- For the "mit" model, it is a vector of length 3 arranged as the intercept, shape parameter and the maximum response.
- For the "schmidt" model, it is a vector of length 3 arranged as scaling parameter, shape parameter (x-value at maximum response) and maximum response.

optim.method

Describes the method used to optimize the model as in the optim() function. The methods include "Nelder-Mead", "BFGS", "CG", "L-BFGS-B", "SANN" and "Brent".

xmin

Numeric value that describes the minimum x value to which the boundary line is to be fitted (default is min(x)).

xmax

A numeric value that describes the maximum x value to which the boundary line is to be fitted (default is max(x)). xmin and xmax determine the subset of the data set used to fit boundary model.

plot

If TRUE, a plot is part of the output. If FALSE, plot is not part of output (default is TRUE).

bp_col

Selects the color of the boundary points.

start

bp_pch Point character as pch of the plot() function. It controls the shape of the boundary points on plot (bp_pch = 16 as default).

bl_col Colour of the boundary line.

lwd Determines the thickness of the boundary line on the plot (default is 1).

line_smooth Parameter that describes the smoothness of the boundary line. (default is 1000).

The higher the value, the smoother the line.

... Additional graphical parameters as in the par() function.

Details

Some inbuilt models are available for the bolides() function. The "explore" option for the argument model generates a plot showing the ocation of the boundary points selected by the binning procedure. This helps to identify which model type is suitable to fit as a boundary line. The suggest model forms are as follows:

1. Linear model ("blm")

$$y = \beta_1 + \beta_2 x$$

where β_1 is the intercept and β_2 is the slope.

2. Linear plateau model ("lp")

$$y = \min(\beta_1 + \beta_2 x, \beta_0)$$

where β_1 is the intercept, β_2 is the slope and β_0 is the maximum response.

3. The logistic ("logistic") and inverse logistic ("inv-logistic") models

$$y = \frac{\beta_0}{1 + e^{\beta_2(\beta_1 - x)}}$$

$$y = \beta_0 - \frac{\beta_0}{1 + e^{\beta_2(\beta_1 - x)}}$$

where β_1 is a scaling parameter, β_2 is a shape parameter and β_0 is the maximum response.

4. Logistic model ("logisticND") (Nelder (1961))

$$y = \frac{\beta_0}{1 + (\beta_1 \times e^{-\beta_2 x})}$$

where β_1 is a scaling parameter, β_2 is a shape parameter and β_0 is the maximum response.

5. Double logistic model ("double-logistic")

$$y = \frac{\beta_{0,1}}{1 + e^{\beta_2(\beta_1 - x)}} - \frac{\beta_{0,2}}{1 + e^{\beta_4(\beta_3 - x)}}$$

where β_1 is a scaling parameter one, β_2 is a shape parameter one, $\beta_{0,1}$ and $\beta_{0,2}$ are the maximum response, β_3 is a scaling parameter two and β_4 is a shape parameter two.

6. Quadratic model ("qd")

$$y = \beta_1 + \beta_2 x + \beta_3 x^2$$

where β_1 is a constant, β_2 is a linear coefficient and β_3 is the quadratic coefficient.

7. Trapezium model ("trapezium")

$$y = \min(\beta_1 + \beta_2 x, \beta_0, \beta_3 + \beta_4 x)$$

where β_1 is the intercept one, β_2 is the slope one, β_0 is the maximum response, β_3 is the intercept two and β_3 is the slope two.

8. Mitscherlich model ("mit")

$$y = \beta_0 - \beta_1 * \beta_2^x$$

where β_1 is the intercept, β_2 is a shape parameter and β_0 is the maximum response.

9. Schmidt model ("schmidt")

$$y = \beta_0 + \beta_1 (x - \beta_2)^2$$

where β_1 is ascaling parameter, β_2 is a shape parameter (x-value at maximum response) and β_0 is the maximum response .

10. Custom model ("other") This option allows you to create your own model form using the function function(). The custom model should be assigned to the argument equation. Note that the parameters for the custom model should be a and b for a two parameter model; a, b and c for a three parameter model; a, b, c and d for a four parameter model and so on.

The function bolides() utilities the optimization procedure of the optim() function to determine the model parameters. There is a tendency for optimization algorithms to settle at a local optimum. To remove the risk of settling for local optimum parameters, it is advised that the function is run using several starting values and the results with the smallest error (residue mean square) can be taken as a representation of the global optimum.

The common errors encountered due to poor start values

- 1. function cannot be evaluated at initial parameters
- 2. initial value in 'vmmin' is not finite

Value

A list of length 5 consisting of the fitted model, equation form, parameters of the boundary line, the residue mean square and the boundary points. Additionally, a graphical representation of the boundary line on the scatter plot is produced.

Author(s)

Chawezi Miti <chawezi.miti@nottingham.ac.uk>

References

Nelder, J.A. 1961. The fitting of a generalization of the logistic curve. Biometrics 17: 89-110.

Phillips, B.F. & Campbell, N.A. 1968. A new method of fitting the von Bertelanffy growth curve using data on the whelk. Dicathais, Growth 32: 317–329.

Schmidt, U., Thöni, H., & Kaupenjohann, M. (2000). Using a boundary line approach to analyze N2O flux data from agricultural soils. Nutrient Cycling in Agroecosystems, 57, 119-129. Schnug, E., Heym, J. M., & Murphy, D. P. L. (1995). Boundary line determination technique (BOLIDES). In P. C. Robert, R. H. Rust, & W. E. Larson (Eds.), site specific management for agricultural systems (p. 899-908). Wiley Online Library.

Examples

cbvn

Fitting boundary line using censored bivariate normal model

Description

This function fits a response model to the upper limits of a scatter plot of of x and y to determine the most efficient response of y as a function of x (given a measurement error of y) based on a censored distribution (Milne et al., 2016). The location of censor in the data cloud is determined based on the maximum likelihood approach. This is done using optimization procedure and hence requires some starting guess parameters for the proposed model. It then compares the results with an uncensored normal bivariate distribution to access the appropriateness of the censored model.

Usage

```
cbvn(data,model="lp", equation=NULL, start, sigh, UpLo="U", optim.method="BFGS", Hessian=FALSE, plot=TRUE, line_smooth=1000, lwd=2, l_col="red",...)
```

Arguments

data	A dataframe with two numeric columns, independent (x) and dependent (y) variables respectively.
model	Selects the functional form of the boundary line. It includes "blm" for linear model, "lp" for linear plateau model, "mit" for the Mitscherlich model, "schmidt" for the Schmidt model, "logistic" for logistic model, "logisticND" for logistic model proposed by Nelder (1961), "inv-logistic" for the inverse logistic model, "double-logistic" for the double logistic model, "qd" for quadratic model and the "trapezium" for the trapezium model. For custom models, set model = "other".
equation	A custom model function writen in the form of an R function. Applies only when argument model="other", else it is NULL.
start	A numeric vector of initial starting values for optimization in fitting the boundary model. Its length and arrangement depend on the suggested model:

• For the "blm" model, it is a vector of length 7 arranged as the intercept, the slope, mean of x, mean of y, standard deviation of x, standard deviation of y and the correlation of x and y.

- For the "1p" model, it is a vector of length 8 arranged as the intercept, the slope, the maximum or plateau response, mean of x, mean of y, standard deviation of x, standard deviation of y and the correlation of x and y.
- For the "mit" model, it is a vector of length 8 arranged as the intercept, shape parameter, the maximum or plateau response, mean of x, mean of y, standard deviation of x, standard deviation of y and the correlation of x and y.
- For the "logistic", "inv-logistic" and "logisticND" models, it is a vector of length 8 arranged as scaling parameter, shape parameter, the maximum or plateau value, mean of x, mean of y, standard deviation of x, standard deviation of y and the correlation of x and y.
- For the "double-logistic" model, it is a vector of length 11 arranged as scaling parameter, shape parameter, maximum response, maximum response, scaling parameter two, shape parameter two, mean of x, mean of y, standard deviation of x, standard deviation of y and the correlation of x and y.
- For the "trapezium" model, it is a vector of length 10 arranged as intercept one, slope one, maximum response, intercept two, slope two, mean of x, mean of y, standard deviation of x, standard deviation of y and the correlation of x and y.
- For the "qd" model, it is a vector of length 8 arranged as a constant, linear coefficient, quadratic coefficient, mean of x, mean of y, standard deviation of x, standard deviation of y and the correlation of x and y.
- For the "schmidt" model, it is a vector of length 8 arranged the scaling parameter, shape parameter (x-value at maximum response), maximum response, mean of x, mean of y, standard deviation of x, standard deviation of y and the correlation of x and y.

sigh Standard deviation of the measurement error.

UpLo Selects the type of boundary. "U" fits the upper boundary and "L" fits the lower boundary.

optim.method Describes the method used to optimize the model as in the optim() function. The methods include "Nelder-Mead", "BFGS", "CG", "L-BFGS-B", "SANN" and

"Brent".

Hessian If True, the hessian matrix is part of the output (default is FALSE').

plot If TRUE, a plot is part of the output. If FALSE, plot is not part of output (default is TRUE).

Parameter that describes the smoothness of the boundary line. (default is 1000). The higher the value, the smoother the line.

lwd Determines the thickness of the boundary line on the plot (default is 1).

1_col Selects the color of the boundary line.

... Additional graphical parameters as in the par() function.

Details

Some inbuilt models are available for the cbvn() function. The suggest model forms are as follows:

1. Linear model ("blm")

$$y = \beta_1 + \beta_2 x$$

where β_1 is the intercept and β_2 is the slope.

2. Linear plateau model ("lp")

$$y = \min(\beta_1 + \beta_2 x, \beta_0)$$

where β_1 is the intercept, β_2 is the slope and β_0 is the maximum response.

3. The logistic ("logistic") and inverse logistic ("inv-logistic") models

$$y = \frac{\beta_0}{1 + e^{\beta_2(\beta_1 - x)}}$$

$$y = \beta_0 - \frac{\beta_0}{1 + e^{\beta_2(\beta_1 - x)}}$$

where β_1 is a scaling parameter , β_2 is a shape parameter and β_0 is the maximum response.

4. Logistic model ("logisticND") (Nelder (1961))

$$y = \frac{\beta_0}{1 + (\beta_1 \times e^{-\beta_2 x})}$$

where β_1 is a scaling parameter, β_2 is a shape parameter and β_0 is the maximum response.

5. Double logistic model ("double-logistic")

$$y = \frac{\beta_{0,1}}{1 + e^{\beta_2(\beta_1 - x)}} - \frac{\beta_{0,2}}{1 + e^{\beta_4(\beta_3 - x)}}$$

where β_1 is a scaling parameter one, β_2 is shape parameter one, $\beta_{0,1}$ and $\beta_{0,2}$ are the maximum response, β_3 is a scaling parameter two and β_4 is a shape parameter two.

6. Quadratic model ("qd")

$$y = \beta_1 + \beta_2 x + \beta_3 x^2$$

where β_1 is a constant, β_2 is a linear coefficient and β_3 is the quadratic coefficient.

7. Trapezium model ("trapezium")

$$y = \min(\beta_1 + \beta_2 x, \beta_0, \beta_3 + \beta_4 x)$$

where β_1 is the intercept one, β_2 is the slope one, β_0 is the maximum response, β_3 is the intercept two and β_3 is the slope two.

8. Mitscherlich model ("mit")

$$y = \beta_0 - \beta_1 * \beta_2^x$$

where β_1 is the intercept, β_2 is a shape parameter and β_0 is the maximum response.

9. Schmidt model ("schmidt")

$$y = \beta_0 + \beta_1 (x - \beta_2)^2$$

where β_1 is a scaling parameter, β_2 is a shape parameter (x-value at maximum response) and β_0 is the maximum response .

The function cbvn() utilities the optimization procedure of the optim() function to determine the model parameters. There is a tendency for optimization algorithms to settle at a local optimum. To remove the risk of settling for local optimum parameters, it is advised that the function is run using several starting values and the results with the smallest likelihood (or AIC) can be taken as a representation of the global optimum.

The common errors encountered due to poor start values

- 1. function cannot be evaluated at initial parameters
- 2. initial value in 'vmmin' is not finite

Value

A list of length 5 consisting of the fitted model, equation form, parameters of the boundary line, AIC (for boundary line model and a null model) and a hessian matrix. Additionally, a graphical representation of the boundary line on the scatter plot is produced.

Author(s)

- 1. Chawezi Miti <chawezi.miti@nottingham.ac.uk>
- 2. Richard Murray Lark <murray.lark@nottingham.ac.uk>

References

Nelder, J.A. 1961. The fitting of a generalization of the logistic curve. Biometrics 17: 89–110.

Lark, R. M., & Milne, A. E. (2016). Boundary line analysis of the effect of water filled pore space on nitrous oxide emission from cores of arable soil. European Journal of Soil Science, 67, 148-159.

Lark, R. M., Gillingham, V., Langton, D., & Marchant, B. P. (2020). Boundary line models for soil nutrient concentrations and wheat yield in national-scale datasets. European Journal of Soil Science, 71, 334-351.

Milne, A. E., Ferguson, R. B., & Lark, R. M. (2006). Estimating a boundary line model for a biological response by maximum likelihood. Annals of Applied Biology, 149, 223–234.

Phillips, B.F. & Campbell, N.A. 1968. A new method of fitting the von Bertelanffy growth curve using data on the whelk. Dicathais, Growth 32: 317–329.

Schmidt, U., Thöni, H., & Kaupenjohann, M. (2000). Using a boundary line approach to analyze N2O flux data from agricultural soils. Nutrient Cycling in Agroecosystems, 57, 119-129.

Examples

evapotranspiration 21

evapotranspiration

Evapotranspiration data

Description

This is a dataset compiled by Sadras & Angus (2006) that is comprises measures of wheat yield and estimated evapotranspiration (ET) from sites in China, the Mediterranean regions of Europe, North America, and Australia. For more details about this dataset refer to Sadras & Angus (2006).

Usage

evapotranspiration

Format

A data.frame with 691 rows and 3 columns:

Region Location of measurement.

ET (mm) Evapotranspiration measurement(mm) measured.

Yield (t/ha) Wheat yield (ton/ha) measured.

Details

This data set should only be used for illustration purposes for this package and should not be used in any form publication without permission from the owners.

Source

Sadras, V. O., & Angus, J. F. (2006). Benchmarking water-use efficiency of rainfed wheat in dry environments. Australian Journal of Agricultural Research, 57, 847-856.

Examples

data(evapotranspiration)

expl_boundary

Testing evidence of boundary existence in dataset

Description

This function determines the probability of having bounding effects in a scatter plot of of x and y based on the clustering of points at the upper edge of the scatter plot (Miti et al.2024). It tests the hypothesis of larger clustering at the upper bounds of a scatter plot against a null bivariate normal distribution with no bounding effect (random scatter at upper edges). It returns the probability (p-value) of the observed clustering given that it a realization of an unbounded bivariate normal distribution.

22 expl_boundary

Usage

```
\label{local-condition} \begin{split} & \text{expl\_boundary}(\texttt{x}, \texttt{y}, \texttt{shells=10}, \texttt{simulations=1000}, \texttt{method="sd-enclidean"}, \texttt{alpha=1}, \\ & \text{plot=TRUE}, \ldots) \end{split}
```

Arguments

A numeric vector of values for the independent variable. Χ A numeric vector of values for the response variable. У shells A numeric value indicating the number of boundary peels (default is 10). simulations The number of simulations for the null bivariate normally distributed data sets used to test the hypothesis (default is 1000). method This describes the measure of boundary points compaction. The methods include "sd-enclidean" for the euclidean distance standard deviation of the each boundary point to the center of data, "Area" for the perimeter around the boundary points and "Perimeter" for the area covering the boundary points. alpha a relative measure of concavity of polygon if method is set to "Area" or "Perimeter". 1 results in a relatively detailed shape, Infinity results in a convex hull. We recommend values in the range of 1 - 5. plot If TRUE, a plot is part of the output. If FALSE, plot is not part of output (default is TRUE).

Details

It is recommended that any outlying observations, as identified by the bagplot() function of the aplpack package are removed from the data. This is also implemented in the simulation step in the expl_boundary() function.

Additional graphical parameters as with the par() function.

Value

A dataframe containing the measures of peel compaction in the left and right sections of the data with their corresponding probability values.

Author(s)

Chawezi Miti <chawezi.miti@nottingham.ac.uk>

References

Eddy, W. F. (1982). Convex hull peeling, COMPSTAT 1982-Part I: Proceedings in Computational Statistics, 42-47. Physica-Verlag, Vienna.

Miti. c., Milne. A. E., Giller. K. E. and Lark. R. M (2024). Exploration of data for analysis using boundary line methodology. Computers and Electronics in Agriculture 219 (2024) 108794.

Park, J.-S. and Oh, S.-J. (2012). A new concave hull algorithm and concaveness measure for n-dimensional datasets. Journal of Information science and engineering, 28(3):587–600.

limfactor 23

Examples

```
x<-evapotranspirationTET(mm) y<-evapotranspirationText{vield(t/ha)} expl_boundary(x,y,10,100) # recommendation is to set simulations to greater than 1000
```

limfactor

Determination of the most limiting factor to biological response

Description

This function determines the most limiting factor based on von Liebig law of the minimum given results of the predicted boundary line values for the different factors of interest. Boundary lines for various factors are fitted and the factor that predicts the minimum response for a particular point is considered as the most limiting factor (Casanova et al. 1995).

Usage

```
limfactor(...)
```

Arguments

... vectors with predicted values from the boundary line models for each factor being evaluated.

Value

A dataframe consisting of the most limiting factor and the minimum predicted response

Author(s)

Chawezi Miti <chawezi.miti@nottingham.ac.uk>

Examples

24 predictBL

red		

Predict boundary response

Description

This function predicts the most efficient response at a level of factor, x, given the parameters of the fitted boundary line.

Usage

```
predictBL(object, x)
```

Arguments

object An output in form of a list from the boundary line fitting using the blqr(),

blbin(), bolides() or cbvn() functions.

x A numeric vector of values for the factor with which response is to be predicted.

Value

A vector predicted value of response.

Author(s)

Chawezi Miti <chawezi.miti@nottingham.ac.uk>

Examples

```
x<-evapotranspiration$`ET(mm)`
y<-evapotranspiration$`yield(t/ha)`
z<-bolides(x,y, start = c(0.5,0.02), model= "blm", xmax = 350)
Results<-predictBL(z,x)
head(Results) # prediction for first 6 lines</pre>
```

soil 25

soil

Soil survey data

Description

This data set is a subset of a data set that was assembled by AgSpace Agriculture Ltd in a soil survey conducted on farms in various management units across England. The survey included measurements of wheat yield as well as various soil parameters. This particular dataset only contains the soil properties pH and phosphorus (P).

Usage

soil

Format

A data.frame with 6110 rows and 3 columns:

```
yield Wheat yield (ton/ha) measured.
```

pH pH measurement.

P soil p (mg/kg).

Details

This data set should only be used for illustration purposes for this package and should not be used in any form publication without permission from the owners.

Examples

data(soil)

SoilP

Soil Phosphorus data

Description

This data set is a subset of a data set that was assembled by AgSpace Agriculture Ltd in a soil survey conducted on farms in various management units across England.

Usage

SoilP

26 start Values

Format

```
A data.frame with 6020 rows and 2 columns:
```

```
yield Wheat yield (ton/ha) measured.
```

P Phosphorus (mg/l).

Examples

```
data(SoilP)
```

SoilpH

Soil pH data

Description

This data set is a subset of a data set that was assembled by AgSpace Agriculture Ltd in a soil survey conducted on farms in various management units across England.

Usage

SoilpH

Format

A data.frame with 6047 rows and 2 columns:

```
yield Wheat yield (ton/ha) measured.
```

pH pH measurement.

Examples

```
data(SoilpH)
```

startValues

Starting values for optimization functions

Description

This functions helps to determine initial values for a selected boundary line model when using the functions blbin(), blqr(), bolides(), cbvn() and ble_profile() to determine model parameters.

Usage

```
startValues(model = "explore", p = NULL, digits = 2, ...)
```

startValues 27

Arguments

model	Selects the functional form of the boundary line. It includes "blm" for linear model, "lp" for linear plateau model, "mit" for the Mitscherlich model, "schmidt" for the Schmidt model, "logistic" for logistic model, "logisticND" for logistic model proposed by Nelder (1961), "inv-logistic" for the inverse logistic model, "double-logistic" for the double logistic model, "qd" for quadratic model, "trapezium" for the trapezium model and "explore" for function use exploration. The default is "explore".
p	The number of selected points used to obtain start values for the logistic mitcherlich and schmidt models. It is NULL for other models.
digits	Number of decimal points for logistic type models (default is 2).
	Additional graphical parameters. Applies to the logistic, mitcherlich and schmidt models to control the text on the plot.

Details

This function uses the locator() function. Once the model is selected, the points that make up the boundary points are selected using mouse click on the plots.

Value

A list containing the parameters of the suggested model.

Author(s)

Chawezi Miti <chawezi.miti@nottingham.ac.uk>

References

Fekedulegn, D., Mac Siurtain, M.P., & Colbert, J.J. 1999. Parameter estimation of nonlinear growth models in forestry. Silva Fennica 33(4): 327–336.

Lark, R. M., & Milne, A. E. (2016). Boundary line analysis of the effect of water filled pore space on nitrous oxide emission from cores of arable soil. European Journal of Soil Science, 67, 148-159.

Examples

startValues(model="explore")

28 summastat

|--|

Description

A function to calculate summary statistics of a set of data.

Usage

```
summastat(x, sigf, varname, plot = TRUE)
```

Arguments

x A vector of numeric values.

sigf The number of significant figures to report (optional).

varname The name of the variable (optional), character so in quotes e.g. "Clay content".

If not used then the variable is called x on plots.

plot If TRUE, a plot is part of the output. If FALSE, plot is not part of output (default

is TRUE).

Value

A matrix containing the mean value, median value, first and third quartiles, sample variance, sample standard deviation, coefficient of skewness, octile skewness, coefficient of kurtosis and the number of probable outliers in a data set. A histogram with a boxplot over it and QQ plot of the variable x if plot=TRUE.

Author(s)

Richard Murray Lark <murray.lark@nottingham.ac.uk>

Examples

```
x<-evapotranspiration$`ET(mm)`
summastat(x,2)</pre>
```

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