

# Orbiter Technical Notes: Planetary axis precession

Martin Schweiger

July 19, 2010

## 1 Introduction

This document describes the implementation of planetary axis precession in Orbiter.

## 2 Definitions

The rotation axis of a celestial body is assumed to rotate around a *precession reference axis* at constant obliquity angle and constant angular velocity. Currently, the orientation of the reference axis (OP) is considered time-invariant and is defined with respect to the ecliptic and equinox of J2000 (see Fig. 1). The axis orientation is defined by the obliquity  $\varepsilon_{\text{ref}}$  (the angle between the axis and the ecliptic north pole,  $N$ ) and the angle from the vernal equinox  $\Upsilon$  to the ascending node of the ecliptic with respect to the body equator,  $L_{\text{ref}}$ . In Orbiter's left-handed system,  $\Upsilon$  is defined as (1,0,0), and  $N$  is defined as (0,1,0). The rotation matrix  $R_{\text{ref}}$  for transforming from ecliptic to precession reference frame is then given by

$$R_{\text{ref}} = \begin{pmatrix} \cos L_{\text{ref}} & 0 & -\sin L_{\text{ref}} \\ 0 & 1 & 0 \\ \sin L_{\text{ref}} & 0 & \cos L_{\text{ref}} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \varepsilon_{\text{ref}} & -\sin \varepsilon_{\text{ref}} \\ 0 & \sin \varepsilon_{\text{ref}} & \cos \varepsilon_{\text{ref}} \end{pmatrix}. \quad (1)$$

The planet's axis of rotation at some time  $t$ , OS, is given relative to the reference axis OP, by obliquity  $\varepsilon_{\text{rel}}$  and longitude  $L_{\text{rel}}$  (see Fig. 2).  $L_{\text{rel}}$  is a linear function of time, and is defined as

$$L_{\text{rel}}(t) = L_0 + 2\pi \frac{t - t_0}{T_p}, \quad (2)$$

where  $t_0$  is a reference date,  $L_0$  is the longitude at that date, and  $T_p$  is the precession period. The rotation from the precession reference frame to the planet's axis frame is described by

$$R_{\text{rel}}(t) = \begin{pmatrix} \cos L_{\text{rel}} & 0 & -\sin L_{\text{rel}} \\ 0 & 1 & 0 \\ \sin L_{\text{rel}} & 0 & \cos L_{\text{rel}} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \varepsilon_{\text{rel}} & -\sin \varepsilon_{\text{rel}} \\ 0 & \sin \varepsilon_{\text{rel}} & \cos \varepsilon_{\text{rel}} \end{pmatrix}. \quad (3)$$

The planet's rotation angle  $\varphi(t)$  is defined via the sidereal period  $T_s$ , and a rotation offset  $\varphi_0$ :

$$\varphi(t) = \varphi_0 + 2\pi \frac{t - t_0}{T_s} + [L_0 - L_{\text{rel}}(t)] \cos \varepsilon_{\text{rel}}, \quad (4)$$

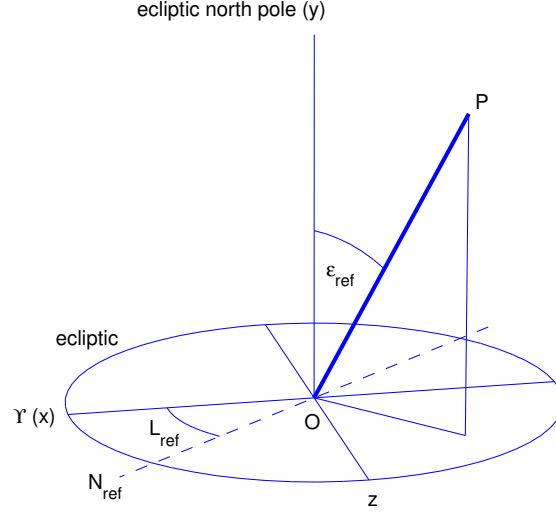


Figure 1: Orientation of the precession reference axis in the ecliptic frame.

where  $t_0$  is a reference time (usually J2000.0). The last term in Eq. 4 accounts for the difference between sidereal and node-to-node rotation period. The rotation is encoded in matrix  $R_{\text{rot}}$ :

$$R_{\text{rot}}(t) = \begin{pmatrix} \cos \varphi & 0 & -\sin \varphi \\ 0 & 1 & 0 \\ \sin \varphi & 0 & \cos \varphi \end{pmatrix}. \quad (5)$$

The full planet transformation is the combination of rotation and precession:

$$R(t) = R_{\text{ref}} R_{\text{rel}}(t) R_{\text{rot}}(t). \quad (6)$$

The direction of the rotation axis is

$$\text{OS} : \mathbf{s}(t) = R(t) \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}. \quad (7)$$

The resulting axis obliquity and longitude of ascending node are

$$\varepsilon_{\text{ecl}}(t) = \cos^{-1} s_y(t), \quad L_{\text{ecl}}(t) = \tan^{-1} \frac{-s_x(t)}{s_z(t)}. \quad (8)$$

Figure 3 shows examples of axis obliquity and longitude of ascending node over one precession cycle for different reference obliquities, as a function of  $L_{\text{rel}}$  (or equivalently, time). Using  $\varepsilon_{\text{ecl}}$  and  $L_{\text{ecl}}$ , an *obliquity matrix*  $R_{\text{ecl}}$  can be defined that rotates from ecliptic to the planet's current precession frame:

$$R_{\text{ecl}}(t) = \begin{pmatrix} \cos L_{\text{ecl}} & 0 & -\sin L_{\text{ecl}} \\ 0 & 1 & 0 \\ \sin L_{\text{ecl}} & 0 & \cos L_{\text{ecl}} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \varepsilon_{\text{ecl}} & -\sin \varepsilon_{\text{ecl}} \\ 0 & \sin \varepsilon_{\text{ecl}} & \cos \varepsilon_{\text{ecl}} \end{pmatrix}. \quad (9)$$

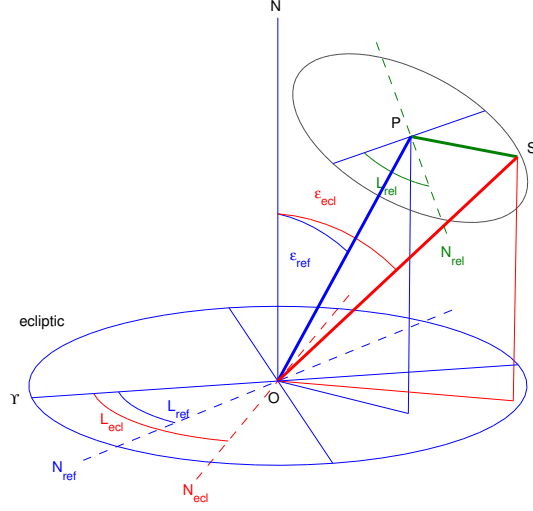


Figure 2: Planet rotation axis.

Note that like  $R_{\text{ref}}R_{\text{rel}}$ , matrix  $R_{\text{ecl}}$  describes a rotation of the axis from ON to OS. However, there is a difference between the rotation around OS. Specifically, the reference axis for  $R_{\text{ecl}}$  is the ascending node of the ecliptic with respect to the planet equator:

$$\text{ON}_{\text{ecl}} : \mathbf{n}(t) = R_{\text{ecl}}(t) \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}. \quad (10)$$

The difference between  $R_{\text{ecl}}$  and  $R_{\text{ref}}R_{\text{rel}}$  can be expressed by an offset matrix  $R_{\text{off}}$ :

$$R_{\text{ecl}}(t)R_{\text{off}}(t) = R_{\text{ref}}R_{\text{rel}}(t), \quad (11)$$

$$R_{\text{off}}(t) = R_{\text{ecl}}^T(t)R_{\text{ref}}R_{\text{rel}}(t). \quad (12)$$

$R_{\text{off}}$  describes a rotation around y, so it has the structure

$$R_{\text{off}}(t) = \begin{pmatrix} \cos \varphi_{\text{off}} & 0 & -\sin \varphi_{\text{off}} \\ 0 & 1 & 0 \\ \sin \varphi_{\text{off}} & 0 & \cos \varphi_{\text{off}} \end{pmatrix}, \quad (13)$$

and the offset angle  $\varphi_{\text{off}}$  is given by

$$\varphi_{\text{off}}(t) = \tan^{-1} \frac{-[R_{\text{off}}]_{13}}{[R_{\text{off}}]_{11}}. \quad (14)$$

Including this offset into the planet's rotation angle leads to an expression for the planet's rotation angle  $r(t)$  with respect to reference direction  $\mathbf{n}(t)$ :

$$r(t) = \varphi(t) + \varphi_{\text{off}}(t). \quad (15)$$

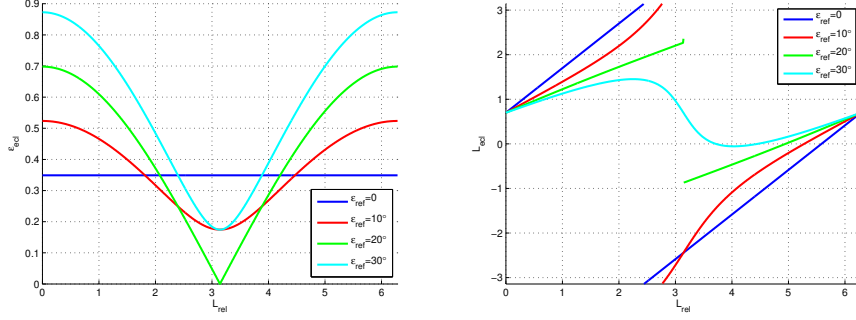


Figure 3: Axis obliquity  $\varepsilon_{\text{ecl}}$  and longitude of ascending node  $L_{\text{ecl}}$  as a function of relative longitude  $L_{\text{rel}}$  over one precession cycle, for four different values of  $\varepsilon_{\text{ref}}$ , and invariant parameters  $\varepsilon_{\text{rel}} = 20^\circ$ ,  $L_{\text{ref}} = 40^\circ$ .

We can now express the full rotation matrix  $R$  defined in Eq. 6, using  $R_{\text{ecl}}$  and  $r$ :

$$R(t) = R_{\text{ecl}}(t)\tilde{R}_{\text{rot}}(t), \quad (16)$$

where

$$\tilde{R}_{\text{rot}}(t) = \begin{pmatrix} \cos r & 0 & -\sin r \\ 0 & 1 & 0 \\ \sin r & 0 & \cos r \end{pmatrix}. \quad (17)$$

### 3 Orbiter interface

#### 3.1 Configuration

The precession and rotation parameters supported in planet configuration files are listed in Table 1. The following default assumptions apply:

- If PrecessionObliquity is not specified,  $\varepsilon_{\text{ref}} = 0$  is assumed. (precession reference is ecliptic normal). In this case, the  $L_{\text{ref}}$  entry is ignored and  $L_{\text{ref}} = 0$  is assumed.
- If PrecessionPeriod is not specified,  $T_p = \infty$  is assumed (rotation axis is stationary).
- If LAN\_MJD is not specified,  $t_0 = 51544.5$  is assumed (J2000.0).
- If LAN is not specified,  $L_0 = 0$  is assumed.
- If Obliquity is not specified,  $\varepsilon_{\text{rel}} = 0$  is assumed.
- If SidRotPeriod is not specified,  $T_s = \infty$  is assumed (no rotation).
- If SidRotOffset is not specified,  $\varphi_0 = 0$  is assumed.

For a retrograde precession of the equinoxes, a negative value of PrecessionPeriod should be used.

## 3.2 API functions

### 3.2.1 void oapiGetPlanetObliquityMatrix (OBJHANDLE hPlanet, MATRIX3 \*mat)

This function returns  $R_{\text{ecl}}(t)$  in Eq. 9 for planet  $h\text{Planet}$  at the current simulation time.

### 3.2.2 double oapiGetPlanetObliquity (OBJHANDLE hPlanet)

This function returns  $\varepsilon_{\text{ecl}}(t)$  in Eq. 8 for planet  $h\text{Planet}$  at the current simulation time.

### 3.2.3 double oapiGetPlanetTheta (OBJHANDLE hPlanet)

This function returns  $L_{\text{ecl}}(t)$  in Eq. 8 for planet  $h\text{Planet}$  at the current simulation time.

### 3.2.4 double oapiGetPlanetCurrentRotation (OBJHANDLE hPlanet)

This function returns the current rotation angle  $r(t)$  in Eq. 15 for planet  $h\text{Planet}$  at the current simulation time.

### 3.2.5 void oapiGetRotationMatrix (OBJHANDLE hPlanet, MATRIX3 \*mat)

This function returns  $R(t)$  in Eq. 6 for planet  $h\text{Planet}$  at the current simulation time.

parameter	config entry
$T_s$	SidRotPeriod [seconds]
$\varphi_0$	SidRotOffset [rad]
$\varepsilon_{\text{rel}}$	Obliquity [rad]
$L_0$	LAN [rad]
$t_0$	LAN_MJD [MJD]
$T_p$	PrecessionPeriod [days]
$\varepsilon_{\text{ref}}$	PrecessionObliquity [rad]
$L_{\text{ref}}$	PrecessionLAN [rad]

Table 1: Rotation and precession parameter entries in planet configuration files.