

# Package ‘TukeyGH77’

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**Type** Package

**Title** Tukey g-&-h Distribution

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**Description** Functions for density, cumulative density, quantile and simulation of Tukey g-and-h (1977) distributions. The quantile-based transformation (Hoaglin 1985 <[doi:10.1002/9781118150702.ch11](https://doi.org/10.1002/9781118150702.ch11)>) and its reverse transformation, as well as the letter-value based estimates (Hoaglin 1985), are also provided.

**License** GPL-2

**Depends** R (>= 4.4.0)

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**Suggests** fitdistrplus

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**Description**

Density, cumulative density, quantile and simulation of the 4-parameter Tukey (1977) *g*-&-*h* distributions. The quantile-based transformation (Hoaglin 1985) and its reverse transformation, as well as the letter-value based estimates (Hoaglin 1985), are also provided.

**Value**

Returned values of individual functions are documented separately.

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**References**

Tukey, J.W. (1977): Modern Techniques in Data Analysis. In: NSF-sponsored Regional Research Conference at Southeastern Massachusetts University, North Dartmouth, MA.

Hoaglin, D.C. (1985): Summarizing shape numerically: The *g*-and-*h* distributions. Exploring data tables, trends, and shapes, pp. 461–513. John Wiley & Sons, Ltd, New York. [doi:10.1002/9781118150702.ch11](https://doi.org/10.1002/9781118150702.ch11)

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GH2z

*Inverse of Tukey g-&-h Transformation*

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**Description**

To transform Tukey *g*-&-*h* quantiles to standard normal quantiles.

**Usage**

GH2z(q, q0 = (q - A)/B, A = 0, B = 1, ...)

**Arguments**

q                    **double vector**, quantiles *q*  
q0                    (optional) **double vector**, standardized quantiles  $q_0 = (q - A)/B$   
A, B                    (optional) **double scalars**, location and scale parameters of Tukey *g*-&-*h* transformation. Ignored if q0 is provided.  
...                    parameters of internal helper function [.GH2z](#)

**Details**

Unfortunately, function `GH2z`, the inverse of Tukey  $g$ -&- $h$  transformation, does not have a closed form and needs to be solved numerically.

For compute intensive jobs, use internal helper function `.GH2z`.

**Value**

Function `GH2z` returns a **double vector** of the same length as input `q`.

**Examples**

```
z = rnorm(1e3L)
all.equal.numeric(.GH2z(z2GH(z, g = .3, h = .1), g = .3, h = .1), z)
all.equal.numeric(.GH2z(z2GH(z, g = 0, h = .1), g = 0, h = .1), z)
all.equal.numeric(.GH2z(z2GH(z, g = .2, h = 0), g = .2, h = 0), z)
```

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 letterValue

*Letter-Value Estimation of Tukey  $g$ -&- $h$  Distribution*


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**Description**

Letter-value based estimation (Hoaglin, 1985) of Tukey  $g$ -,  $h$ - and  $g$ -&- $h$  distribution. All equation numbers mentioned below refer to Hoaglin (1985).

**Usage**

```
letterValue(
  x,
  g_ = seq.int(from = 0.15, to = 0.25, by = 0.005),
  h_ = seq.int(from = 0.15, to = 0.35, by = 0.005),
  halfSpread = c("both", "lower", "upper"),
  ...
)
```

**Arguments**

<code>x</code>	<b>double vector</b> , one-dimensional observations
<code>g_</code>	<b>double vector</b> , probabilities used for estimating $g$ parameter. Or, use <code>g_ = FALSE</code> to implement the constraint $g = 0$ (i.e., an $h$ -distribution is estimated).
<code>h_</code>	<b>double vector</b> , probabilities used for estimating $h$ parameter. Or, use <code>h_ = FALSE</code> to implement the constraint $h = 0$ (i.e., a $g$ -distribution is estimated).
<code>halfSpread</code>	<b>character</b> scalar, either to use 'both' for half-spreads (default), 'lower' for half-spread, or 'upper' for half-spread.
<code>...</code>	additional parameters, currently not in use

**Details**

Unexported function `letterV_g()` estimates parameter  $g$  using equation (10) for  $g$ -distribution and the equivalent equation (31) for  $g$ -&- $h$  distribution.

Unexported function `letterV_B()` estimates parameter  $B$  for Tukey  $g$ -distribution (i.e.,  $g \neq 0$ ,  $h = 0$ ), using equation (8a) and (8b).

Unexported function `letterV_Bh_g()` estimates parameters  $B$  and  $h$  when  $g \neq 0$ , using equation (33).

Unexported function `letterV_Bh()` estimates parameters  $B$  and  $h$  for Tukey  $h$ -distribution, i.e., when  $g = 0$  and  $h \neq 0$ , using equation (26a), (26b) and (27).

Function `letterValue` plays a similar role as `fitdistrplus::start.arg.default`, thus extends `fitdistrplus::fitdist` for estimating Tukey  $g$ -&- $h$  distributions.

**Value**

Function `letterValue` returns a 'letterValue' object, which is **double vector** of estimates  $(\hat{A}, \hat{B}, \hat{g}, \hat{h})$  for a Tukey  $g$ -&- $h$  distribution.

**Note**

Parameter `g_` and `h_` does not have to be truly unique; i.e., `all.equal` elements are allowed.

**References**

Hoaglin, D.C. (1985). Summarizing Shape Numerically: The  $g$ -and- $h$  Distributions. [doi:10.1002/9781118150702.ch11](https://doi.org/10.1002/9781118150702.ch11)

**Examples**

```
set.seed(77652); x = rGH(n = 1e3L, g = -.3, h = .1)
letterValue(x, g_ = FALSE, h_ = FALSE)
letterValue(x, g_ = FALSE)
letterValue(x, h_ = FALSE)
(m3 = letterValue(x))

library(fitdistrplus)
fit = fitdist(x, distr = 'GH', start = as.list.default(m3))
plot(fit) # fitdistrplus:::plot.fitdist
```

**Description**

Density, distribution function, quantile function and simulation for Tukey  $g$ -&- $h$  distribution with location parameter  $A$ , scale parameter  $B$ , skewness  $g$  and elongation  $h$ .

**Usage**

```
dGH(x, A = 0, B = 1, g = 0, h = 0, log = FALSE, ...)

rGH(n, A = 0, B = 1, g = 0, h = 0)

qGH(p, A = 0, B = 1, g = 0, h = 0, lower.tail = TRUE, log.p = FALSE)

pGH(q, A = 0, B = 1, g = 0, h = 0, lower.tail = TRUE, log.p = FALSE, ...)
```

**Arguments**

x, q	<b>double vector</b> , quantiles
A	<b>double</b> scalar, location parameter $A = 0$ by default
B	<b>double</b> scalar, scale parameter $B > 0$ . Default $B = 1$
g	<b>double</b> scalar, skewness parameter $g = 0$ by default (i.e., no skewness)
h	<b>double</b> scalar, elongation parameter $h \geq 0$ . Default $h = 0$ (i.e., no elongation)
log, log.p	<b>logical</b> scalar, if TRUE, probabilities $p$ are given as $\log(p)$ .
...	other parameters of function <code>vuniroot2</code>
n	<b>integer</b> scalar, number of observations
p	<b>double vector</b> , probabilities
lower.tail	<b>logical</b> scalar, if TRUE (default), probabilities are $Pr(X \leq x)$ otherwise, $Pr(X > x)$ .

**Value**

Function `dGH` returns the density and accommodates **vector** arguments A, B, g and h. The quantiles x can be either **vector** or **matrix**. This function takes about 1/5 time of `gk::dgh`.

Function `pGH` returns the distribution function, only taking scalar arguments and **vector** quantiles q. This function takes about 1/10 time of function `gk::pgh`.

Function `qGH` returns the quantile function, only taking scalar arguments and **vector** probabilities p.

Function `rGH` generates random deviates, only taking scalar arguments.

**Examples**

```
(x = c(NA_real_, rGH(n = 5L, g = .3, h = .1)))
dGH(x, g = c(0,.1,.2), h = c(.1,.1,.1))

p0 = seq.int(0, 1, by = .2)
(q0 = qGH(p0, g = .2, h = .1))
range(pGH(q0, g = .2, h = .1) - p0)

q = (-2):3; q[2L] = NA_real_; q
(p1 = pGH(q, g = .3, h = .1))
range(qGH(p1, g = .3, h = .1) - q, na.rm = TRUE)
(p2 = pGH(q, g = .2, h = 0))
```

```
range(qGH(p2, g = .2, h = 0) - q, na.rm = TRUE)
curve(dGH(x, g = .3, h = .1), from = -2.5, to = 3.5)
```

vuniroot2

*Vectorised One Dimensional Root (Zero) Finding***Description**

To solve a monotone function  $y = f(x)$  for a given **vector** of  $y$  values.

**Usage**

```
vuniroot2(
  y,
  f,
  interval = stop("must provide a length-2 `interval`"),
  tol = .Machine$double.eps^0.25,
  maxiter = 1000L
)
```

**Arguments**

<code>y</code>	<b>numeric vector</b> of $y$ values
<code>f</code>	monotone <b>function</b> $f(x)$ whose roots are to be solved
<code>interval</code>	<b>length-2 numeric vector</b>
<code>tol</code>	<b>double</b> scalar, desired accuracy, i.e., convergence tolerance
<code>maxiter</code>	<b>integer</b> scalar, maximum number of iterations

**Details**

Function `vuniroot2`, different from `vuniroot`, does

- accept `NA_real_` as element(s) of  $y$
- handle the case when the analytic root is at lower and/or upper
- return a root of `Inf` (if  $\text{abs}(f(\text{lower})) \geq \text{abs}(f(\text{upper}))$ ) or `-Inf` (if  $\text{abs}(f(\text{lower})) < \text{abs}(f(\text{upper}))$ ), when the function value  $f(\text{lower})$  and  $f(\text{upper})$  are not of opposite sign.

**Value**

Function `vuniroot2` returns a **numeric vector**  $x$  as the solution of  $y = f(x)$  with given **vector**  $y$ .

**Examples**

```

library(rstpm2)

# ?rstpm2::vuniroot does not accept NA \eqn{y}
tryCatch(vuniroot(function(x) x^2 - c(NA, 2:9), lower = 1, upper = 3), error = identity)

# ?rstpm2::vuniroot not good when the analytic root is at `lower` or `upper`
f <- function(x) x^2 - 1:9
vuniroot(f, lower = .99, upper = 3.001) # good
tryCatch(vuniroot(f, lower = 1, upper = 3, extendInt = 'no'), warning = identity)
tryCatch(vuniroot(f, lower = 1, upper = 3, extendInt = 'yes'), warning = identity)
tryCatch(vuniroot(f, lower = 1, upper = 3, extendInt = 'downX'), error = identity)
tryCatch(vuniroot(f, lower = 1, upper = 3, extendInt = 'upX'), warning = identity)

vuniroot2(c(NA, 1:9), f = function(x) x^2, interval = c(1, 3)) # all good

```

z2GH

*Tukey g-&-h Transformation***Description**

To transform standard normal quantiles to Tukey *g*-&-*h* quantiles.

**Usage**

```
z2GH(z, A = 0, B = 1, g = 0, h = 0)
```

**Arguments**

*z* [double](#) scalar or [vector](#), standard normal quantiles.  
*A, B, g, h* [double](#) scalar or [vector](#), parameters of Tukey *g*-&-*h* distribution

**Details**

Function [z2GH](#) transforms standard normal quantiles to Tukey *g*-&-*h* quantiles.

**Value**

Function [z2GH](#) returns a [double](#) scalar or [vector](#).

**Note**

Function `gk:::z2gh` is not fully vectorized, i.e., cannot take [vector](#) *z* and [vector](#) *A/B/g/h*, as of 2023-07-20 (package `gk` version 0.6.0)

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